

Availability Modeling of Single Unit System Subject to Degradation Post Repair After Complete Failure Using RPGT When Repair is Perfect

Dr. Ritikesh Kumar*

Extension Lecturer, Mathematics Department, Government Girls College, Sec 14, Gurugram, Haryana, India.
Corresponding Author Email: kunduritikesh199@gmail.com*



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ABSTRACT

In this paper we intend to analysis the behavioral of a single unit system having degradation after complete failure using RPGT. In initially the given system unit is working at full capacity which may have two types of failures, one is direct and another is through partial failure. There is a single repairman who repairs the unit on each failure. Problem is formulated and solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables.

Keywords: Availability; Reliability; Primary Circuits; Secondary Circuits; Tertiary Circuits; Degraded State; RPGT; Mean Time to System Failure.

1. Introduction

The world reliability is used in our daily life. In current daye the technology without reliability is unthinkable. In thi paper we are discussed the behavior analysis of single unit system degradation. It is a continuous process of instruments and component of system subject to operating and repair mechanism. Fuzzy concept is used to determine failure/working state of system unit. Consider various probabilities, a transition diagram of the given system is developed to analysis Primary, Secondary, Tertiary and Base state. Problem is analysis and solved using RPGT. Repair and Failure are statistically independent. Mean time to system failure, availability, and number of server visits and busy period of the server are analysis to study the behavior of given system for steady state. Particular cases are taken to study the effect of failure and repair rates on mean time to system failure, availability, and expected number of server visits and busy period of the server. Profit optimization is also discussed. System behavior is discussed with the help of graphs and tables.

1.1. Assumptions and Notations

1. A single repair man is available.
2. The failure times and repair times are exponential.
3. Repair is imperfect.
4. When the system is in failed state then Nothing can fail.

Table 1.1

Vertex i	Primary Circuits (CL1)	Secondary Circuits (CL2)
0	(0,1,2)	Nil
1	(1,2,0)	Nil
2	(2,0,1)	Nil

Transition Diagram of the system is given in Figure 1.1.

$$S_0 = A, \quad S_1 = \bar{A}, \quad S_2 = a$$

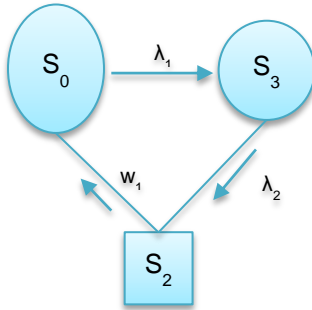


Figure 1.1

In the above table 1.1 we analysis that at working state ‘0’ which have maximum number of primary circuits, so state ‘0’ is the base state.

Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State ‘0’)

Table 1.2

Vertex j	$(0 \xrightarrow{S_r} j): (P_0)$	(P1)
0	$(0 \xrightarrow{S_1} 0): \{0,1,0\}$	Nil
1	$(0 \xrightarrow{S_1} 1): \{0,1\}$	Nil
2	$(3 \xrightarrow{S_1} 2): \{0,1,2\}$	Nil

Transition Probabilities

Table 1.3

$q_{ij}^{(t)}$	$P_{ij} = q_{ij}^{*(t)}$
$q_{0,1} = \lambda_1 e^{-\lambda_1 t}$	$p_{0,1} = 1$
$q_{1,2} = \lambda_2 e^{-\lambda_2 t}$	$p_{1,2} = 1$
$q_{2,0} = w_1 e^{-w_1 t}$	$p_{2,0} = 1$

Mean Sojourn Times

Table 1.4

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0^{(t)} = e^{-\lambda_1 t}$	$\mu_0 = 1/\lambda_1$
$R_1^{(t)} = e^{-\lambda_2 t}$	$\mu_1 = 1/\lambda_2$
$R_2^{(t)} = e^{-w_1 t}$	$\mu_2 = 1/w_1$

1.2. Evaluation of Parameters

The Mean time of the given system failure and all the key parameters of the system under steady state conditions are evaluated, applying RPGT and '0' as the base-state of the system as under:

$$V_{0,0} = (0,1,2,0)$$

$$= p_{0,1} p_{1,2} p_{2,0} = 1$$

$$V_{0,1} = (0,1)$$

$$= p_{0,1} = 1$$

$$V_{0,2} = (0,1,2)$$

$$= p_{0,1} p_{1,2} = 1$$

$$\mathbf{MTSF} (T_0) = \left[\sum_{i,SR} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} i \right) \right\} \mu_i}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[1 - \sum_{SR} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr(sff)} \xi \right) \right\}}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$= (V_{0,0} \mu_0 + V_{0,1} \mu_1) / (V_{0,0} \mu_0 + V_{0,1} \mu_1 + V_{0,2} \mu_2)$$

1.3. Availability of the System

The regenerative states at which the system is available are 'j' = 0, 1 and the regenerative states are 'i' = 0, 1, 2 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow j}) \right\} f_j \mu_j}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow i}) \right\} \mu_i^1}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j \right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1 \right]$$

$$= (V_{0,0} f_0 \mu_0 + V_{0,1} f_1 \mu_1) / (V_{0,0} \mu_0 + V_{0,1} \mu_1 + V_{0,2} \mu_2)$$

Proportional Busy Period of the Server:

$$B_0 = \left[\sum_{j,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow j}) \right\} n_j}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow i}) \right\} \mu_i^1}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$= \left[\sum_j V_{\xi,j}, n_j \right] \div \left[\sum_i V_{\xi,i}, \mu_i^1 \right]$$

$$= (V_{2,0} \mu_2) / (V_{0,0} \mu_0 + V_{0,1} \mu_1 + V_{0,2} \mu_2)$$

Expected Number of Inspections by the repair man:

$$V_0 = \left[\sum_{j,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow j}) \right\}}{\prod_{k_1 \neq \xi} \left\{ 1 - V_{k_1 k_1} \right\}} \right\} \right] \div \left[\sum_{i,SR} \left\{ \frac{\left\{ \text{pr}(\xi^{SR \rightarrow i}) \right\} \mu_i^1}{\prod_{k_2 \neq \xi} \left\{ 1 - V_{k_2 k_2} \right\}} \right\} \right]$$

$$V_0 = \left[\sum_j V_{\xi,j} \right] \div \left[\sum_i V_{\xi,i}, \mu_i^1 \right]$$

$$V_0 = (V_{2,0} \mu_2) / (V_{0,0} \mu_0 + V_{0,1} \mu_1 + V_{0,2} \mu_2)$$

Illustration: When failure and repair rates are equal

$$MTSF (T_0) = [2w/(2w+\lambda)]$$

MTSF Table

Table 1.5

T_0	$w = 0.80$	$w = 0.90$	$w = 1.00$
$\lambda = 0.50$	0.761904	0.782609	0.800000
$\lambda = 0.60$	0.727272	0.750000	0.769230
$\lambda = 0.70$	0.695652	0.720000	0.740740

MTSF Graph

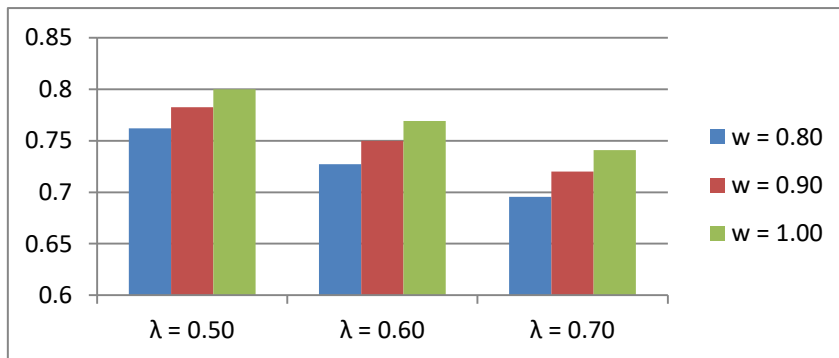


Figure 1.2

Availability of the System (A_0)

Availability of the System Table

Table 1.6

A_0	$w = 0.80$	$w = 0.90$	$w = 1.00$
$\lambda = 0.50$	0.761904	0.782609	0.800000
$\lambda = 0.60$	0.727272	0.750000	0.769230
$\lambda = 0.70$	0.695652	0.720000	0.740740

Availability of the System Graph

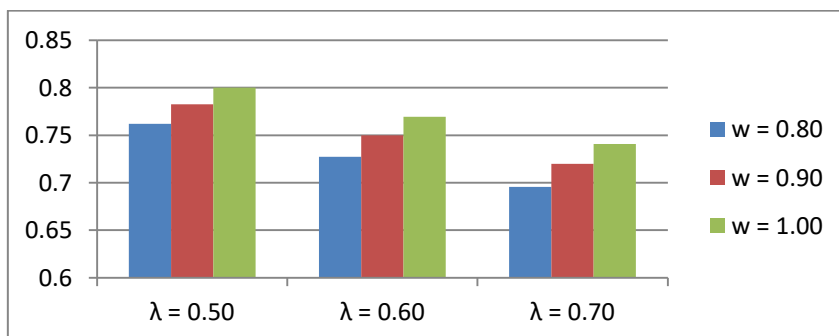


Figure 1.3

Fractional Busy Period of the Server (B_0) in Unit Time:

$$(B_0) = [\lambda / (2w + \lambda)]$$

Busy Period of the Server Table

Table 1.7

B_0	$w = 0.80$	$w = 0.90$	$w = 1.00$
$\lambda = 0.50$	0.238095	0.217391	0.200000
$\lambda = 0.60$	0.272727	0.250000	0.230769
$\lambda = 0.70$	0.304348	0.280000	0.259259

Busy Period of the Server Graph

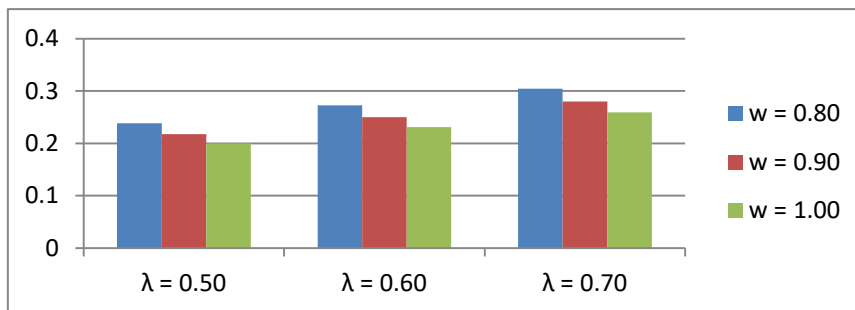


Figure 1.4

Expected Fractional Number of Server's Visits (V_0) in Unit Time:

Expected Number of Server's Visits Table

Table 1.8

V_0	$w = 0.80$	$w = 0.90$	$w = 1.00$
$\lambda = 0.50$	0.238095	0.217391	0.200000
$\lambda = 0.60$	0.272727	0.250000	0.230769
$\lambda = 0.70$	0.304348	0.280000	0.259259

Expected Number of Server's Visits Graph

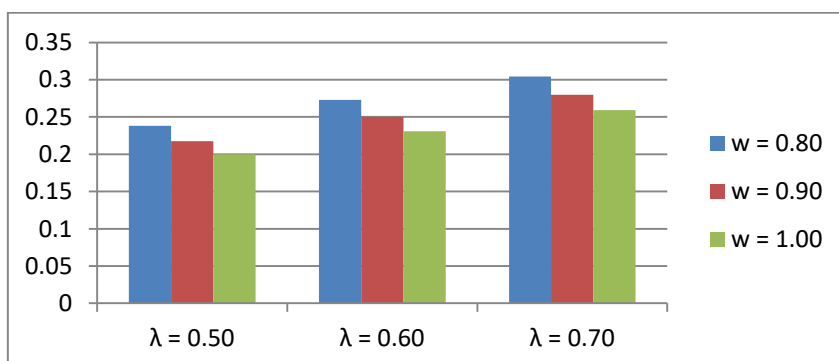


Figure 1.5

Profit Function

$$= A_0R_0 - (B_0R_1 + V_0R_2)$$

$$= A_0R_0 - B_0R_1 - V_0R_2$$

Where

A_0 = Availability of System

B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man

R_0 = Revenue

R_1 = Busy Period per Unit

R_2 = Per Visit Cost

$$R_0 = 1000$$

$$R_1 = 50$$

$$R_2 = 100$$

$$\text{Profit} = [2000w - 150\lambda] / [(2w + \lambda)]$$

Profit Table 1.9

	w = 0.5	w = 0.6	w = 0.7
$\lambda = 0.5$	726.1905	750.0000	770.0000
$\lambda = 0.6$	686.3636	712.5000	734.6154
$\lambda = 0.7$	598.0000	678.0000	701.8519

Profit Function Graph

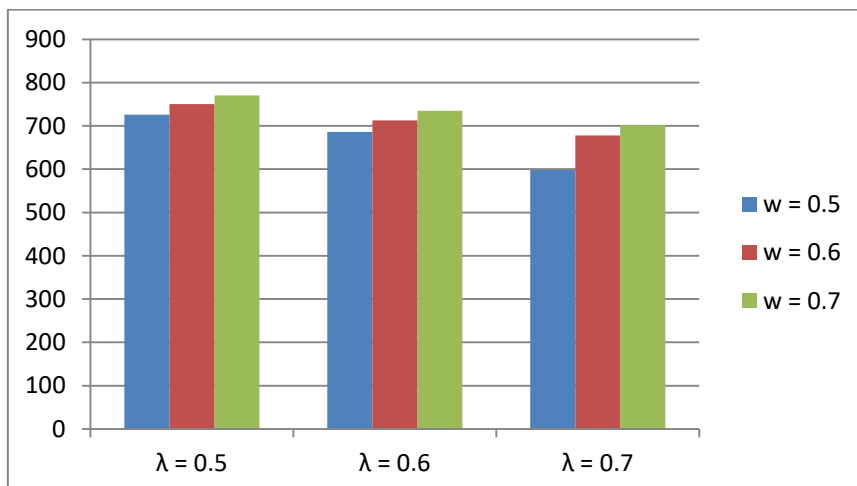


Figure 1.6

2. Conclusion

From analysis of above tables and graph we conclude that the finding obtained using RPGT is similar as obtained by using RPT and other techniques. Here, we determined the results easily and quickly without writing any state equations and without any lengthy procedures, long calculations and simplification. If we want the optimum values of system parameter and we confirm that the repair rates to achieve the optimum values for various system parameters and profit function I also maximize. Finding Results in corollary having same value with the results obtained by any researchers.

Declarations

Source of Funding

This study did not receive any grant from funding agencies in the public or not-for-profit sectors.

Conflict of Interest

The author declares that he has no conflict of interest.

Consent for Publication

The author declares that he consented to the publication of this study.

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